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INSTABILITY OF FLOW IN A STREAMWISE CORNER

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ABSTRACT

The linear stability of an incompressible laminar flow in the blending boundary layer between the boundary layer in a 90^o streamwise corner and a Blasius boundary layer well away from the corner is examined using a locally parallel flow approximation. It is shown that the influence of the outer boundary conditions associated with oblique modes of disturbances which are anti-symmetric about the bisector plane have a profound effect on the stability of the flow. As a result, in good agreement with observation, the critical streamwise Reynolds number, associated with a spanwise location is significantly reduced as the corner is approached, being $R_{cr} = 60$ approximately for spanwise distance of $z^* = 6x^*R^{-1}$ from the corner compared with $R_{cr} = 322$ approximately for $z^* = 20x^*R^{-1}$, where x^* measures downstream distance from the leading edges. At $R = 600$, the growth rate of the most amplified mode of disturbance at the former location is over six times greater than that at the latter; the corresponding wave angle at the two locations is respectively 44^o and 5^o, approximately.

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1. Introduction

The secondary flow which arises in the streamwise corners of a wind tunnel has important bearing on the size of the available test section. In experiments involving laminar flow in a two-dimensional boundary layer, for example, the role of the side walls of the tunnel in the observed transition is not always clear. Further the spread of the secondary flow from the junction of a wing with the fuselage of an aircraft has implications for laminar flow control on wings.

A knowledge of the nature of the flow in a streamwise corner and its stability characteristics is therefore of considerable practical importance. A model flow which has been studied is that in which the corner is formed by two semi-infinite plane surfaces inclined at an angle, usually a right angle, to each other with the free stream flow along the corner (figure 1). The basic flow has a similarity solution which is symmetric about the bisector plane (see Rubin(1966), for example) such that well away from the corner it matches the two-dimensional Blasius boundary layer flow along the plane surfaces while in the corner it is governed by fully three-dimensional equations of motion. In between these regions, Rubin(1966) identified a 'blending boundary layer' where the influence of the corner decays algebraically away from the corner. Further, Rubin showed that to leading order the spanwise velocity profile in this region exhibits a reverse flow in the boundary layer.

Zamir(1981) estimated from experimental evidence that in the absence of pressure gradient, transition in a right angled streamwise corner occurs at a value of the square root of the Reynolds number, based on the streamwise distance, of $R = 100$ (henceforth R will be referred to as the Reynolds number). Unlike the experimental results of El-Gamal and Barclay(1978), Zamir and Young (1979) found breakdown of similarity in the velocity profiles. Zamir(1981) attributed this as a development of flow instability.

Lakin and Hussaini (1984) derived the equations governing small perturbations of the basic flow in the 'blending' boundary layer identified by Rubin using a locally parallel flow approximation. They considered heuristic solutions of these equations based on the critical layer in the boundary layer.

In this paper, we re-consider the stability of the blending boundary layer for a right angled streamwise corner, using a locally parallel flow approximation as done by Lakin and Hussaini and show that the instability is dominantly driven by the outer boundary conditions associated with disturbances which are, unlike the basic flow, anti-symmetric about the bisector plane. The spanwise cross-flow turns out to be of too insignificant a magnitude to account for the observed early transition in the flow, unlike in the case of flow past a swept wing, for example.

In section 2, the equations of motion and the boundary conditions are given. The equations obtained are exactly those given by Lakin and Hussaini (1984) although it has not been necessary to make any a priori assumptions about the nature of the disturbances. The equation governing the normal component of the perturbed velocity satisfy a modified Orr-Sommerfeld equation. Since the basic flow is symmetric about the bisector plane, the outer boundary conditions need to be applied on the bisector plane rather than at 'infinity'. This gives rise to the possibility of modes of disturbances which are symmetric or anti-symmetric about the bisector plane. It is found that the anti-symmetric modes are the most unstable.

The method of orthonormalization is used to determine the eigenvalues of the stability problem and is briefly described in section 3.

The results of the investigation are presented in section 4. As expected, it is found that the critical Reynolds number, R_{cr} decreases with decrease in spanwise distance from the corner, it being given by $R_{cr} = 54$ for a spanwise distance of $z^* = 6R^{-1}x^*$ compared with $R = 322$ for $z^* = 20R^{-1}x^*$; the growth rate of the the most unstable disturbance at $R = 600$ is over six times greater at the former location than that at the the latter while the corresponding wave-angle is over nine times greater. Further, it is found that in the proximity of the corner, the three-dimensional modes of disturbance are unstable at a lower Reynolds number than that for two-dimensional ones.

2. Basic Equations and boundary conditions

We define Cartesian co-ordinates (x^*, y^*, z^*) as shown in figure 1, with the two semi-infinite rigid planes intersecting at right angle in the line Ox^*. A viscous incompressible fluid of uniform density flows along the corner with the undisturbed velocity away from the planes given by $(U^*, 0, 0)$ where U^* is a constant. The undisturbed flow is symmetric about the bisector plane $y^* = z^*$ so that it is only necessary to consider the flow in the region between this bisector plane and the rigid plane $y^* = 0$, say, with appropriate boundary conditions on the two planes. The fluid velocity satisfies the exact non-dimensional equations ($\underline{u}^* = U^*\underline{u}(x, y, z)$, $t^* = U^{*-1}lt$, $\underline{x}^* = l\underline{x}, p^* = \rho^* U^{*2} p$ where l denotes the downstream distance of the location under consideration):

$$\frac{\partial \underline{u}}{\partial t} + \underline{u}.\nabla\underline{u} \;=\; -\nabla p + \frac{1}{R^2}\nabla^2\underline{u} \tag{1}$$

and

$$\nabla.\underline{u} \;=\; 0, \tag{2}$$

where $R = (U^* l/\nu^*)^{1/2}$ is the Reynolds number. The boundary conditions to be satisfied are

$$\underline{u}(x,0,z,t) = 0, \quad \underline{u}(x,\infty,\infty,t) = (1,0,0), \tag{3}$$

together with appropriate boundary condition on the bisector plane $x = z$.

We consider a small perturbation about a basic steady flow, so that

$$\underline{u} = \overline{\underline{u}}(\underline{x}) + \tilde{\underline{u}}(\underline{x},t), \;\; p = \overline{p}(\underline{x}) + \tilde{p}(\underline{x},t). \tag{4}$$

On substituting (4) into (1) and respectively equating zeroth order and first order terms in $\tilde{\underline{u}}$ and $\tilde{p}$ to zero we have

$$\overline{\underline{u}}.\nabla\overline{\underline{u}} + \nabla\overline{p} \;=\; \frac{1}{R^2}\nabla^2\overline{\underline{u}} \tag{5}$$

$$\frac{\partial\tilde{\underline{u}}}{\partial t} + \overline{\underline{u}}.\nabla\tilde{\underline{u}} + \tilde{\underline{u}}.\nabla\overline{\underline{u}} \;=\; -\nabla p + \frac{1}{R^2}\nabla^2\tilde{\underline{u}} \tag{6}$$

$$\nabla.\overline{\underline{u}} = 0, \quad \nabla.\tilde{\underline{u}} = 0 \tag{7}$$

with, writing $\overline{\underline{u}} = (\overline{U}, \overline{V}, \overline{W})$,

$$\begin{gathered}\overline{\underline{u}}(x,0,z) = 0, \;\; \overline{\underline{u}}(x,\infty,\infty) = (1,0,0) \\ \frac{\partial\overline{U}}{\partial y} = \frac{\partial\overline{U}}{\partial z}, \;\; \overline{V} = \overline{W} \text{ on } y = z\end{gathered} \tag{8}$$

and

$$\tilde{\underline{u}}(x,0,z,t) = 0, \;\; \tilde{\underline{u}}(x,\infty,\infty) = 0 \tag{9}$$

together with appropriate boundary conditions on $\tilde{\underline{u}}$ at $y = z$.

In region $\delta < z << x$, where δ is the non-dimensional thickness of a Blasius boundary layer, $\delta = 4.9x^{1/2}R^{-1}$, Rubin (1966) has shown that for a steady flow, symmetric about the plane $y = z$, (5) has an approximate similarity solution,

$$\overline{\underline{u}} = (f'(\eta) + O(R^{-1}\frac{z}{x}), R^{-1}\overline{v}(\eta), R^{-1}(\overline{w}(\eta) + O(\frac{z^2}{x^2}))) \; + \; O(R^{-2}) \tag{10}$$

where a prime denotes differentiation with respect to the dependent variable $\eta = yR/\sqrt{x}$ and $\overline{v} = \frac{1}{2}(\eta f' - f)$, $\overline{w} = a_0 g(\eta)$ and f and g satisfy

$$f''' + \frac{1}{2}ff'' = 0; \; f(0) = f'(0) = 0; \; f'(\infty) = 1 \tag{11}$$

$$g'' + \frac{1}{2}(fg)' = \frac{1}{2}; \; g(0) = 0; g(\infty) = -1 \tag{12}$$

with $a_0 = .86039$. Here the streamwise pressure gradient is $o(R^{-1})$. Thus the approximate basic flow corresponds to a Blasius boundary layer flow with an imposed cross-flow, $R^{-1}\overline{w}$.

associated with the normal component of velocity in the boundary layer on the other rigid plane.

We consider the location $x = 1(x^* = l)$, $z = R^{-1}z_0$ ($z_0 > 5$), and introduce the concept of locally parallel flow. Without loss of generality, we consider a disturbance centered on $(1, y, R^{-1}z_0)$ on an appropriate scale of $O(R^{-1})$. We write

$$x = 1 + R^{-1}x_1,\ y = R^{-1}\eta_1 + O(R^{-2}),\ z = R^{-1}(z_0 + z_1),\ t = R^{-1}t_1 \tag{13}$$

and note that $\eta = \eta_1 + O(R^{-1})$, $' \equiv \frac{d}{d\eta_1}$, $\nabla = R\nabla_1$ etc. Thus locally, in the vicinity of $(1, y, R^{-1}z_0)$,

$$\underline{\overline{u}} = (f'(\eta_1), R^{-1}\overline{v}(\eta_1), R^{-1}\overline{w}(\eta_1))\ +\ O(R^{-2}). \tag{14}$$

If we write $\underline{\tilde{u}} = (u, v, w)$, then on substituting (14) and (15) into (6) and (7), we obtain after some manipulation,

$$[(\mathcal{L} + \frac{1}{R}\overline{v}')\nabla_1^2 - f'''\frac{\partial}{\partial x_1} - \frac{1}{R}\overline{w}''\frac{\partial}{\partial z_1}]v\ =\ 0, \tag{15}$$

$$\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial \eta_1} + \frac{\partial w}{\partial z_1}\ =\ 0, \tag{16}$$

$$\mathcal{L}\zeta\ + f''\frac{\partial v}{\partial z_1} - \frac{1}{R}w'\frac{\partial v}{\partial x_1}\ =\ 0 \tag{17}$$

where $\zeta = \frac{\partial u}{\partial z_1} - \frac{\partial w}{\partial x_1}$ is the normal component of perturbed vorticity and

$$\mathcal{L}\ =\ \frac{\partial}{\partial t_1} + \underline{\overline{u}}.\nabla_1 - \frac{1}{R^2}{\nabla_1}^2. \tag{18}$$

Equations (15)-(17) have a solution of the form

$$[v(x,y,z;t), \zeta(x,y,z;t)]\ =\ [v_1(\eta_1), \zeta_1(\eta_1)]e^{i(\alpha x_1 + \beta z_1 - \alpha c t_1)} \tag{19}$$

where v_1 and ζ_1 satisfy the de-coupled equations:

$$[(D^2 - \lambda^2)^2 - (\overline{v}D + \overline{v}' + iR(\alpha f' + \frac{\beta}{R}\overline{w} - \alpha c))(D^2 - \lambda^2) + iR(\alpha f''' + \frac{\beta}{R}\overline{w}'')]v_1 = 0 \tag{20}$$

$$[iR(\alpha f' + \frac{\beta}{R}\overline{w} - \alpha c) + \overline{v}D - (D^2 - \lambda^2)]\zeta_1 + iR(\beta f'' - \frac{\alpha}{R}\overline{w}')]v_1 = 0. \tag{21}$$

Here $D \equiv d/dz_1$ and $\lambda^2 =\ \alpha^2 + \beta^2$. Equation (20) corresponds to the Orr-Sommerfeld equation for a basic flow $\overline{U} = \overline{u} + \frac{\beta}{\alpha R}\overline{w}$; it is usual to drop the terms involving $\overline{v}$and$\overline{v}'$, but these are retained here consistent with retaining $R^{-1}\overline{w}$ in the basic flow. Retaining these terms makes minor changes in the stability characteristics. Equation (20) is the same as given by Lakin and Hussaini (1984) although in deriving it, it has not been necessary to make any a priori assumption about the nature of the disturbance.

Boundary conditions

The approximate basic flow in (20) is independent of the spanwise location z and it is tempting to obtain (as is usually done in the case of the Orr-Sommerfeld problem associated with the Blasius boundary layer) the outer boundary conditions from (9) and from the limiting form as $\eta_1 \to \infty$ of (20)and apply these at a large value of η_1; the limiting form of the fourth order equation (20) has two exponentially growing solutions and two exponentially decaying solutions, one solution in each category being viscous and the other inviscid in character. Care, however, is required in the present case since the basic flow is regarded as being symmetric about the bisector plane $y = z$ so that the outer boundary condition must strictly be applied at this plane, instead of at some large value of η_1 as the restriction $4.9 < z_0 << R$ imply that the normal to the plane $y = 0$ at the spanwise location $z = R^{-1}z_0$ meets the bisector plane at $\eta_1 \equiv \eta_{1out} = z_0$, where $4.9 < \eta_{1out} << R$. It is through this requirement that the dependence of the stability characteristics on the spanwise location enters into the problem.

Although the basic flow is symmetric about the plane $y = z$, no such restriction applies to the perturbations. Accordingly, the perturbation velocity $\underline{\tilde{u}} = (u, v, w)$ may satisfy one of four possible basic sets of boundary conditions on $y = z$: (i)$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial z}$, $v = w$,(ii) $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial z}$, $v = -w$, (iii) $u = 0$, $v = w$, (iv) $u = 0$, $v = -w$. Conditions (i) and (ii) imply that u is symmetric while $v - w$ is respectively anti-symmetric and symmetric in $y = z$. Conditions (iii) and (iv) imply that u is anti-symmetric while $v - w$ is respectively anti-symmetric and symmetric in $y = z$. It is found that the most unstable mode of disturbance corresponds to condition (iii). We therefore consider only this case in detail here; the other cases may be considered similarly without any difficulty. In view of the continuity equation, it follows from condition (iii) that we must have

$$(D + i\beta)v \;=\; 0 \text{ on } \eta_1 = z_0 \tag{22}$$

Next, we note that in spite of the restriction on its range, η_{1out} still lies outside the boundary layer so that we may consider the limiting form of (20) as $\eta_1 \to z_0 >> 1$ in determining the second outer boundary condition. Hence,

$$(D^2 - \lambda^2)(D - \chi)v \;=\; 0 \text{ on } \eta_1 = z_0 >> 1 \tag{23}$$

where $\chi = \frac{1}{2}(\overline{v}(\infty) - (\overline{v}^2(\infty) + 4(\lambda^2 + iR(\alpha + \beta\overline{w}(\infty)/R)))^{1/2})$. The choice (23) means that of the four solutions to the limiting form of (20) described above, we only discard the viscous exponentially growing solution, the growth rate of this solution being much greater than the

corresponding inviscid solution. The conditions at the surface $\eta_1 = 0$ are

$$v_1(0) = Dv_1(0) = 0 \tag{24}$$

3. Numerical method

The linear equation (20) together with the boundary conditions (22)-(24) is solved using the technique of orthonormalization (see for example, Davey (1973)) which allows the use of standard shooting methods. We first express (20) as a system of first order ordinary differential equations:

$$\phi_i{}' = \sum_{j=1}^{4} a_{ij}\phi_j, \quad (i = 1, ...4) \tag{25}$$

where

$$\begin{aligned} &\phi_1 = v_1, \quad \phi_2 = \phi_1', \quad \phi_3 = \phi_2' - \lambda^2\phi_1 \\ &\phi_4 = \phi_3', \end{aligned} \tag{26}$$

and the non-zero elements of the matrix a_{ij} are:

$$\begin{gathered} a_{12} = a_{23} = a_{34} = 1, \; a_{21} = \lambda^2 \\ a_{41} = -iR(\alpha f''' + \beta\overline{w}''/R) \\ a_{43} = \lambda^2 + iR(\alpha f' + \beta\overline{w}/R - \alpha c) + \overline{v}', \quad a_{44} = \overline{v}. \end{gathered} \tag{27}$$

The boundary conditions (22)-(24) take the form:

$$\begin{aligned} \phi_2(z_0) + i\beta\phi_1(z_0) &= 0 \\ \phi_4(z_0) - \chi\phi_3(z_0) &= 0 \\ \phi_1(0) = \phi_2(0) &= 0 \end{aligned} \tag{28}$$

Equations (25) are to be integrated from $\eta_1 = z_0$ for various values of z_0 to $\eta_1 = 0$. The steady flow, characterized by f', $\overline{v}$ and $\overline{w}$, is evaluated at $2M$ points by integrating (14)-(17) using fourth order Runge-Kutta integration.

Corresponding to the two outer boundary conditions given in (28), two independent solutions satisfying these conditions and the four equations (25) can be chosen. We denote these by vectors $\underline{\phi}^{(j)}$, $(j = 1, 2)$ where

$$\underline{\phi}^{(j)} = [\phi_1^{(j)}, \phi_2^{(j)}, \phi_3^{(j)}, \phi_4^{(j)}]^T$$

with starting values at $\eta_1 = z_0$ given by:

$$\underline{\phi}^{(1)} = [1, -i\beta, 0, 0]^T$$

$$\underline{\phi}^{(2)} = [0, 0, 1, \chi]^T \tag{29}$$

With these initial values the system (25) is integrated using M-point fourth-order Runge-Kutta integration. The general solution is a linear combination of these two solutions:

$$\underline{\phi} = b_1\underline{\phi}^{(1)} + b_2\underline{\phi}^{(2)} \tag{30}$$

where b_i are chosen to satisfy the two remaining boundary conditions at $z = 0$. Since the boundary conditions at $z = 0$ are homogeneous, this requires that the determinant of a certain 2×2 matrix, whose elements are the appropriate elements of the vectors $\underline{\phi}^{(j)}(j = 1, 2)$, vanish.

However, since the eigenvalue problem is stiff, rounding errors cause the base solutions $\underline{\phi}^{(j)}$ to lose their linear independence fairly quickly during the integration. To circumvent this difficulty, the base vectors are orthonormalized every ten integration steps, say (a total of $(2M/10) - 1$ orthonormalizations since no orthonormalization is performed at the final integration step). Thus, every ten integration steps, $\underline{\phi}^{(j)}$ is replaced by $\underline{\overline{\phi}}^{(j)}$ where

$$\underline{\overline{\phi}}^{(1)} = \underline{\phi}^{(1)}/|\underline{\phi}^{(1)}|$$

$$\underline{\overline{\phi}}^{(2)} = [\underline{\phi}^{(2)} - (\underline{\phi}^{(2)}.\underline{\overline{\phi}}^{(1)})\underline{\overline{\phi}}^{(1)}]/|[\;]| \tag{31}$$

with $|[\;]|$ in the denominator denoting the modulus of the bracketed numerator. In satisfying the boundary conditions at $\eta_1 = 0$, the determinant of the corresponding 2×2 matrix, whose elements are now the appropriate elements of $\underline{\overline{\phi}}^{(j)}$, $(j = 1, 2)$ is required to vanish. If the eigenfunctions are needed, it is necessary to keep an account of all the orthnormalizations. However, to determine the eigenvalues it is only necessary to solve iteratively the non-linear equation,

$$f(\alpha, \beta, R, c) \;=\; 0 \tag{32}$$

corresponding to the vanishing determinant at $\eta_1 = 0$. This is done here using Muller's method.

4. Results

We have restricted consideration to solutions corresponding to the most unstable disturbances. The neutral curves, which represent the projections onto the $\alpha - R$ plane of the contours of the neutral surface, given by $\beta =$ constant, corresponding to three values

of the spanwise stations z_0 ($z^* = lR^{-1}z_0$ with $x^* = l$) are shown in figure 2(a)- (c). Contours are plotted for values of β in the range $0 \le \beta \le \beta_{cr}$, where β_{cr} is that value of β which corresponds to the critical Reynolds number, R_{cr}. As is expected, proximity to the corner has a destabilizing effect. In the cases shown, instability to three-dimensional disturbances in the above range of spanwise wavenumber occurs at a lower Reynolds number than it does for a two-dimensional disturbance, this feature being more pronounced at stations closer to the corner than those away from it. Proximity to the corner also has a destabilizing effect on the two-dimensional mode of disturbance ($\beta = 0$). It may be noted that at $z_0 = 6$, terms neglected in the basic flow in (14) may become important and inclusion of these terms will modify the stability characteristics at this location. However, we expect that the characteristics depicted in figure 2(c) will still be approximately correct. For values of $z_0 < 6$ approximately, we expect that it will be necessary to consider the exact, fully three-dimensional basic flow.

It is interesting that the instability is driven by the outer boundary conditions, the most unstable disturbance having a finite value of β. This is consistent with Dhanak (1981) where it was shown that spanwise waviness in one of the walls of a channel has a destabilizing effect on the plane Poiseuille flow through the channel. The effect is much more pronounced in the present case in view of the presence of the other wall.

The neutral curves for $\beta = \beta_{cr}$ associated with four spanwise stations are shown in figure 3 for comparison. The neutral curve for a Blasius boundary layer ($\beta_{cr} = 0$) is also included in the figure. (Note, however, that in this case, contrary to normal practice, the $O(R^{-1})$ normal component of the basic flow is retained in the associated Orr-Sommerfeld equation, consistent with (20); this has the effect of shifting the neutral curve for this case slightly towards higher Reynolds number.) The critical Reynolds number decreases and the range of the unstable streamwise and spanwise wavenumbers increases as the corner is approached. The values of the critical parameters are given in Table 1. The values of the critical Reynolds number of 60 for $z_0 = 6$ and 110 for $z_0 = 7.5$ are consistent with the experimental prediction of transitional Reynolds number of $R = 100$ by Zamir (1981). The value of β_{cr} increases while α_{cr} decreases, so that the critical wave-angle $\epsilon_{cr} = \tan^{-1}(\beta_{cr}/\alpha_{cr})$ increases, as the corner is approached;$\epsilon_{cr} = 48^o$ for $z_0 = 6$ compared with $\epsilon_{cr} = 5^o$ for $z_0 = 20$. As can be seen from figure 3, the neutral curves for finite values of z_0 approach the neutral curve for the Blasius boundary layer as z_0 becomes large.

It is expected that the most unstable mode of disturbance will dominate the initial development of the instability. The growth rate of the most unstable disturbance is plotted as a function of the Reynolds number for each spanwise station z_0 in figure 4. At $z_0 = 6$ the growth rate increases from $\alpha c_i = 0$ at $R = 54$ to $\alpha c_i = 11.3 \times 10^{-3}$ at $R = 600$. At $R = 600$

the growth rate of the disturbance is over six times greater at $z_0 = 6$ than it is at $z_0 = 20$.

The variation of the wave-angle $\epsilon = \tan^{-1}(\beta/\alpha)$ corresponding to the most unstable mode of disturbance with Reynolds number is shown in figure 5. The value of this wave-angle increases as the corner is approached; at $R = 600$, $\epsilon = 44^o$ for $z_0 = 6$ compared with $\epsilon = 5^o$ at $z_0 = 20$. ϵ decreases at first as R is decreased from its value of 600, but then increases somewhat as the critical Reynolds number is approached, the final increase being most significant for the case $z_0 = 6$; $\epsilon = 48^o$ at the critical Reynolds number for $z_0 = 6$.

5. Conclusions

It is shown that the instability of the flow in the vicinity of a streamwise right angled corner is dominantly driven by oblique disturbances which are anti-symmetric in the bisector plane. The magnitude of the cross flow is found to be too small to be a significant factor in the observed early transition of this flow. It is found that close to the corner, three-dimensional disturbances in a particular range of spanwise wavenumbers are unstable at a lower Reynolds number than two-dimensional disturbances. Close to the corner, the critical Reynolds number has a value of around $R \sim 50$ compared with $R \sim 300$ for the Blasius boundary layer far away from the corner. This is consistent with the experimental prediction (Zamir, 1981) of transitional Reynolds number of $R = 100$ for this flow. At $R = 600$ the growth rate of the most unstable disturbance is over six times greater at a distance $z^* = 6R^{-1}x^*$ than it is at $z^* = 20R^{-1}x^*$ while the corresponding wave-angle is over nine times greater.

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z_0	R_{cr}	α_{cr}	β_{cr}	$(\alpha c_r)_{cr}$	ϵ_{cr}
6	53.95	0.1223	0.1360	0.0616	48.0
7.5	115.27	0.1283	0.1015	0.0553	38.3
10	210.90	0.1449	0.0806	0.0590	29.1
20	321.91	0.1736	0.0141	0.0684	4.6
∞	324.43	0.1741	0.	0.0685	0.

Table 1: Critical values of the stability parameters. The values in the last row are for a Blasius boundary layer and are evaluated using the corresponding Orr-Sommerfeld equation in which terms propotional to $\overline{v}$, $\overline{v'}$ are retained.

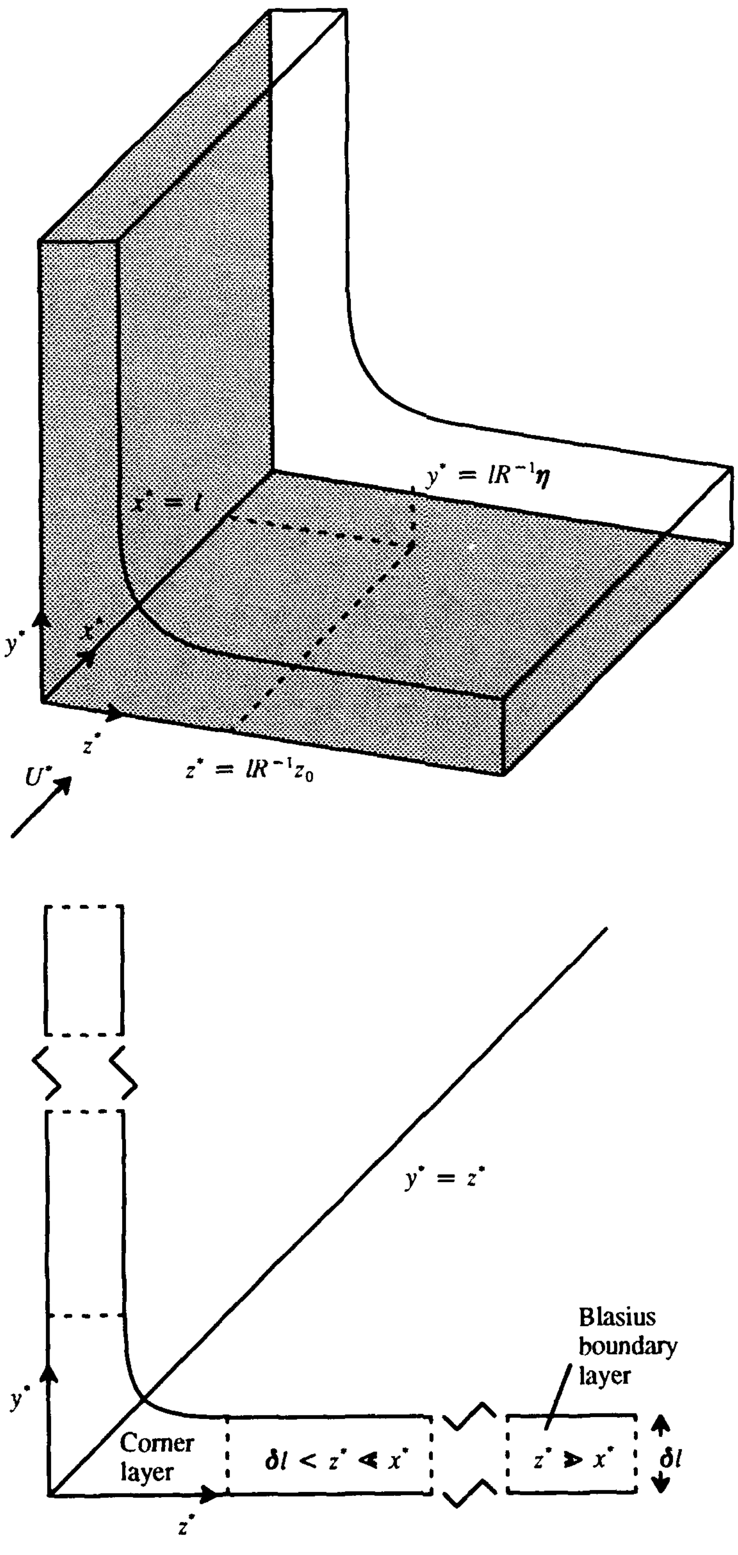

Figure 1. Schematics of corner flow geometry

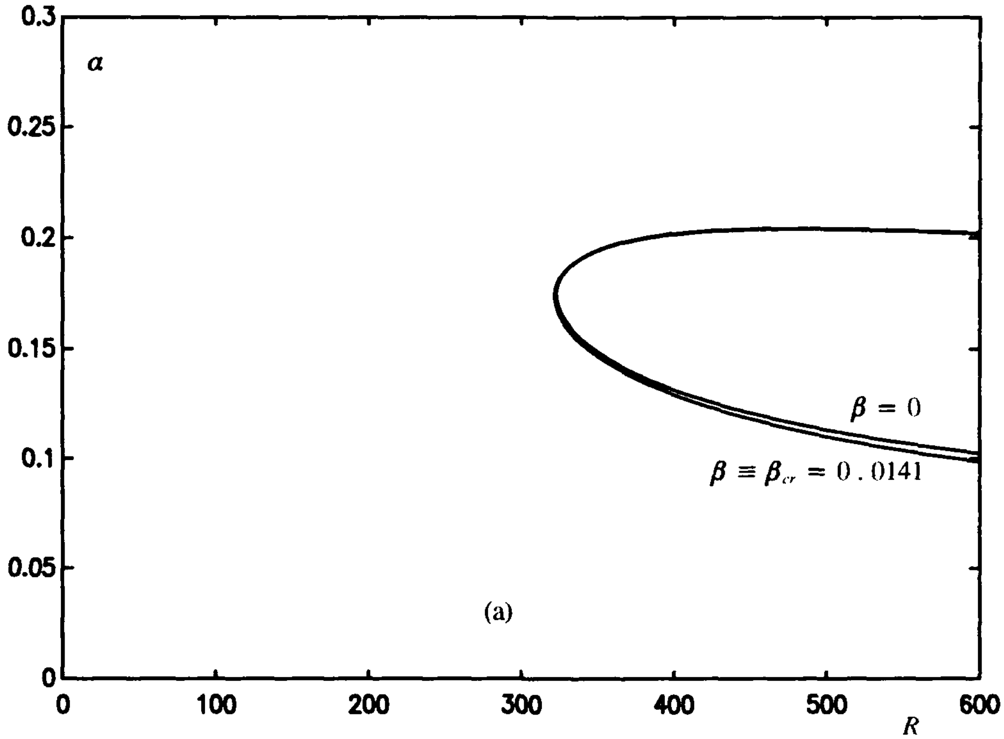

Figure 2. Neutral curves, represented by contours of the spanwise wavenumber β = constant in the $\alpha - R$ plane, at spanwise locations (a) $z_0 = 20$, (b) $z_0 = 10$, (c) $z_0 = 6$. Contours are shown for β in the range $0 \leq \beta \leq \beta_{cr}$ where β_{cr} is the value of β corresponding to the critical Reynolds number. Here spanwise distance $z^* = x^* R^{-1} z_0$. .

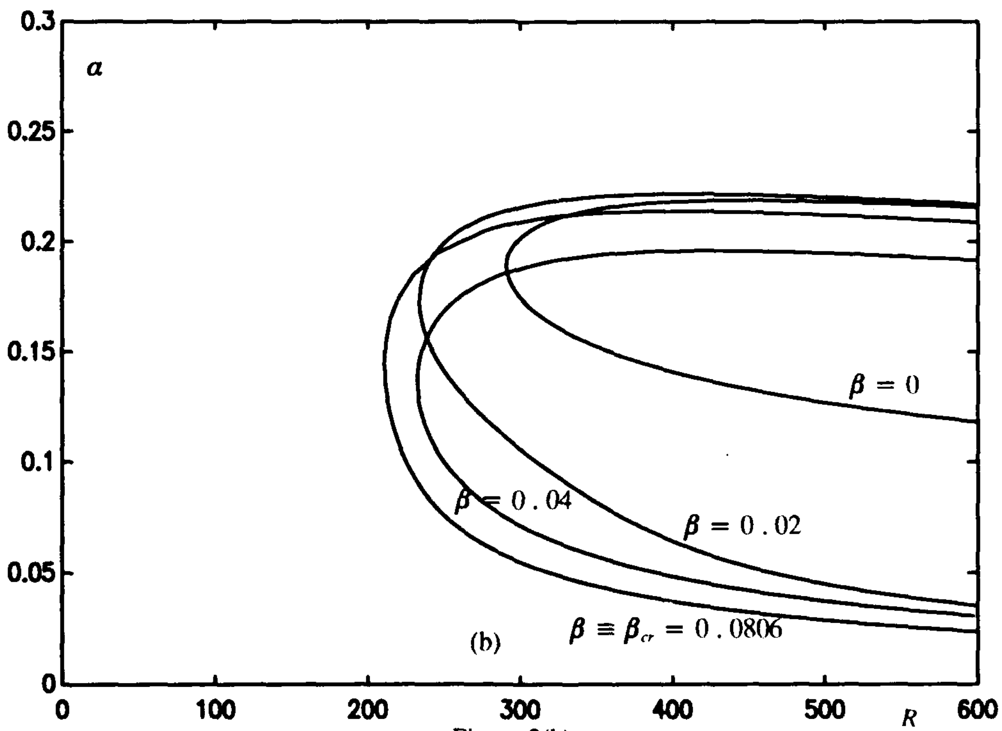

Figure 2(b).

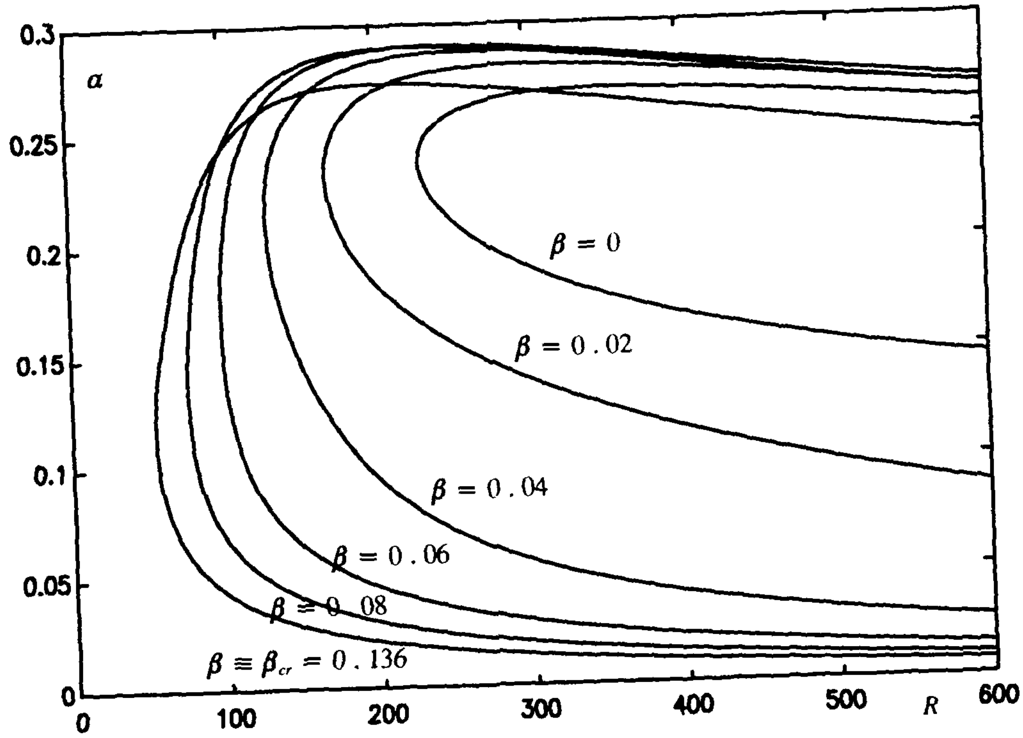

Figure 2(c).

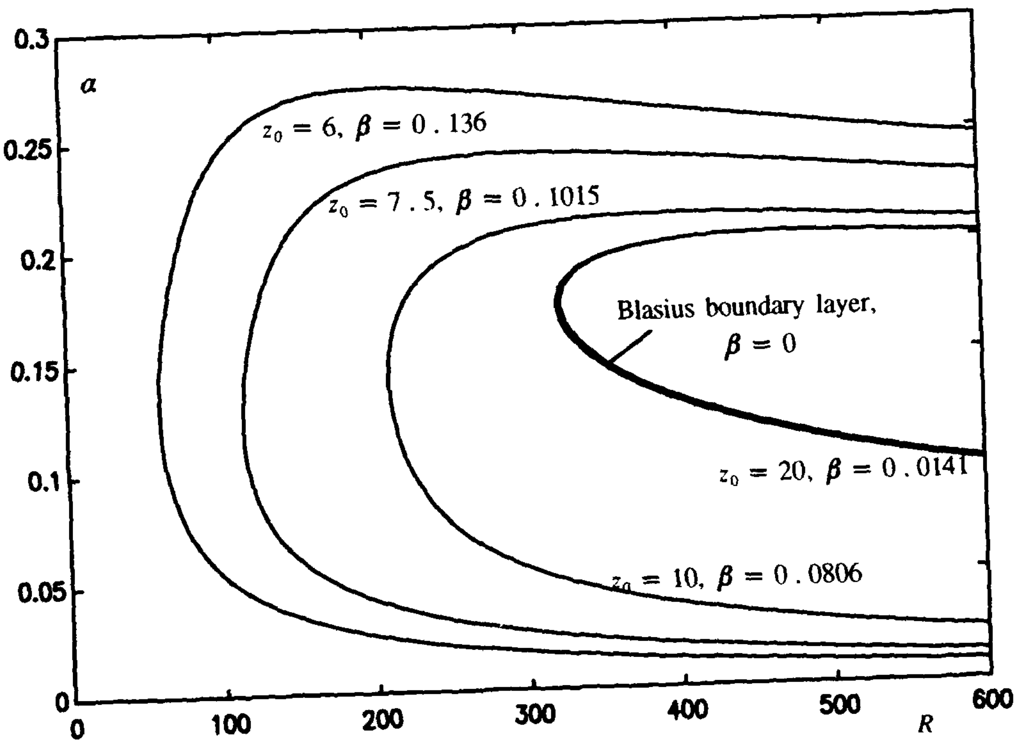

Figure 3. Variation of the neutral curve contour, corresponding to $\beta = \beta_{cr}$, with spanwise location z_0 The associated critical value of β is shown on each curve and the neutral curve corresponding to a Blasius boundary layer is included for comparison.

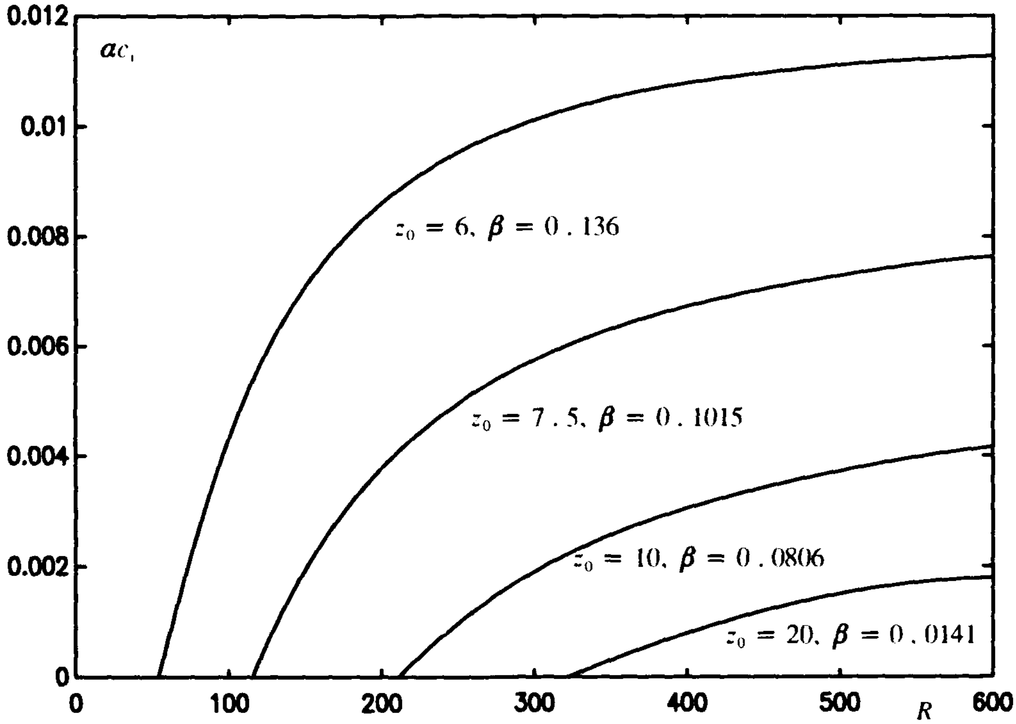

Figure 4. Growth–rate of most unstable disturbance for various spanwise locations z_0 as a function of Reynolds number R. The fixed value of β corresponding to maximum growth rate in each case is shown on the associated curve.

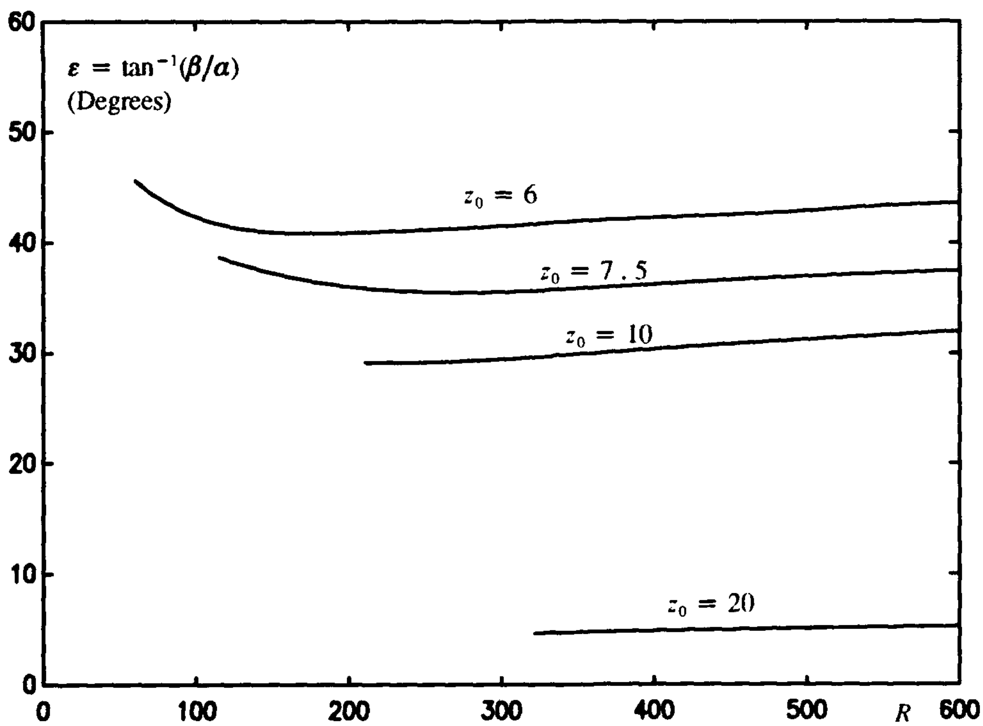

Figure 5. Wave–angle for the most unstable disturbance as a function of the Reynolds number at various spanwise locations z_0 . The corresponding wave–angle is zero for a Blasius boundary layer flow.

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The linear stability of an incompressible laminar flow in the blending boundary layer between the boundary layer in a 90^{o} streamwise corner and a Blasius boundary layer well away from the corner is examined using a locally parallel flow approximation. It is shown that the influence of the outer boundary conditions associated with oblique modes of disturbances which are anti-symmetric about the bisector plane have a profound effect on the stability of the flow. As a result, in good agreement with observation, the critical streamwise Reynolds number, associated with a spanwise location is significantly reduced as the corner is approached, being $R_{cr} = 60$ approximately for spanwise distance of $z^* = 6x^*R^{-1}$ from the corner compared with $R_{cr} = 322$ approximately for $z^* = 20x^*R^{-1}$, where x^* measures downstream distance from the leading edges. At R=600, the growth rate of the most amplified mode of disturbance at the former location is over six times greater than that at the latter; the corresponding wave angle at the two locations is respectively 44^{o} and 5^{o}, approximately.

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